Section 4 – 1: Classifying Triangles

Parts of a Triangle:

Triangle – a three-sided polygon

Name –
Sides –
Vertices –
Angles –

Classifying Triangles by Angles:

Acute Δ          Obtuse Δ          Right Δ

Equiangular Δ -

Classifying Triangles by Sides:

Scalene Δ          Isosceles Δ          Equilateral Δ
**Example #1:** Identify the indicated type of triangle in the figure.

a.) isosceles triangles

b.) scalene triangles

**Example #2:** Find $x$ and the measure of each side of equilateral triangle $RST$.

**Example #3:** Find $x$, $JM$, $MN$, and $JN$ if $\triangle JMN$ is an isosceles triangle with $JM \cong MN$. 
Section 4 – 1: Classifying Triangles
In-Class Worksheet

1.) Identify the indicated types of triangles.
   a.) right b.) isosceles
c.) scalene d.) obtuse

2.) Find \(x\) and the measure of each side of the triangle.
   a.) \(\triangle ABC\) is equilateral with \(AB = 3x - 2\), \(BC = 2x + 4\), and \(CA = x + 10\).

   b.) \(\triangle DEF\) is isosceles, \(\angle D\) is the vertex angle, \(DE = x + 7\), \(DF = 3x - 1\), and \(EF = 2x + 5\).

3.) Describe each triangle by as many of the following words as apply: acute, obtuse, right, scalene, isosceles, or equilateral.
   a.)
   b.)
   c.)
Section 4 – 2: Angles of Triangles

Notes

Angle Sum Theorem:
- The sum of the measures of the angles of a \( \text{____________} \) is \( \text{______} \).

Example #1: Find the missing angle measures.

a.)

b.)

Third Angle Theorem:
- If two angles of one triangle are \( \text{____________} \) to two angles of a second triangle, then the third angles of the triangles are \( \text{____________} \).
Exterior Angle Theorem:

- An exterior angle is formed by one side of a ______________ and the extension of another ________.
- Remote interior angles are the angles of a triangle that are not ______________ to a given ______________ angle.
- The measure of an exterior angle of a triangle is ___________ to the sum of the measures of the two _________________ interior angles.

![Diagram of a triangle with exterior angles labeled]

Example #2: Find the measure of each of the following angles.

a.) 

b.)

\[ m \angle A = \] 

\[ m \angle DCB = \]
Section 4 – 6: Isosceles Triangles
Notes

**Isosceles Triangle**: A triangle with at least _________ sides congruent.

The angle formed by the congruent sides is called the _________ angle.

The two angles formed by the base and one of the congruent sides are called _________ angles.

**Isosceles Triangle Theorem**: If two sides of a triangle are ________________, then the angles opposite those sides are ________________.

**Ex:**

**Example #1**: If $\overline{DE} \cong \overline{CD}$, $\overline{BC} \cong \overline{AC}$, and $m\angle CDE = 120$, what is the measure of $\angle BAC$?
**Theorem 4.10:** If two angles of a ___________ are congruent, then the sides opposite those angles are _____________.

Ex:

**Example #2:**

a.) Name all of the congruent angles.

b.) Name all of the congruent segments.

**Corollary 4.3:** A triangle is ________________ if and only if it is ________________.

**Corollary 4.4:** Each angle of an equilateral triangle measures ________.

**Example #3:** $\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$.

a.) Find $m\angle 1$ and $m\angle 2$.

b.) Find $x$. 

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Section 4 – 3: Congruent Triangles

Notes

**Congruent Triangles:** triangles that are the same ________ and ________

- Each triangle has three __________ and three ________.  
- If all _______ of the corresponding parts of two triangles are ________________, then the triangles are ________________.

Congruent Triangles:

Corresponding Congruent Angles:

Corresponding Congruent Sides:

**Definition of Congruent Triangles (CPCTC):**

- Two triangles are congruent if and only if their corresponding parts are _________________.
- **CPCTC** – Corresponding parts of congruent triangles are congruent
Example #1: In the following figure, $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$, $TV = 24$, and $RV = 23$.

Name the corresponding congruent angles and sides.

Name the congruent triangles.

Properties of Triangle Congruence:

<table>
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<th>Reflexive</th>
<th>Symmetric</th>
<th>Transitive</th>
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<tbody>
<tr>
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<td><img src="image2" alt="Symmetric" /></td>
<td><img src="image3" alt="Transitive" /></td>
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Example #2: If $\triangle WZX \cong \triangle STJ$, name the congruent angles and congruent sides.

Angles -

Sides -
Section 4 - 4: Proving Congruence – SSS, SAS

Notes

Side-Side-Side Congruence: If the __________ of one triangle are congruent to the sides of a second triangle, then the triangles are ______________.

Abbreviation:

Side-Angle-Side Congruence: If two sides and the included __________ of one triangle are congruent to two __________ and the included angle of another triangle, then the triangles are ______________.

Abbreviation:

Example #1: Write a proof.

Given: \( EI \cong FH \), \( FE \cong HI \), and G is the midpoint of both \( EI \) and \( FH \).

Prove: \( \triangle FEG \cong \triangle HIG \)
Example #2: Write a proof.

\textbf{Given:} \overline{DE} \text{ and } \overline{BC} \text{ bisect each other.}

\textbf{Prove:} \triangle DGB \cong \triangle EGC

Example #3: Write a proof.

\textbf{Given:} \overline{AB} \cong \overline{AC} \text{ and } \overline{BY} \cong \overline{CY}

\textbf{Prove:} \triangle BYA \cong \triangle CYA
Proving Triangle Congruence

Side-Side-Side and Side-Angle-Side Congruence

Write a two-column proof for the following problems.

1.) Given: \( EF \cong GH \)
\( FG \cong HE \)

Prove: \( \triangle EFG \cong \triangle GHE \)

2.) Given: \( \overline{PQ} \) bisects \( \angle SPT \)
\( SP \cong TP \)

Prove: \( \triangle SPQ \cong \triangle TPQ \)
3.) Given: \( \overline{AC} \cong \overline{GC} \)
\( \overline{EC} \) bisects \( \overline{AG} \)

Prove: \( \triangle GEC \cong \triangle AEC \)
Section 4 – 5: Proving Congruence – ASA, AAS

Notes

Angle–Side–Angle Congruence: If two ___________ and the included _________ of one triangle are congruent to two angles and the included side of another triangle, then the triangles are _________________.

Abbreviation:

Angle–Angle–Side Congruence: If two angles and a non-included side of one triangle are congruent to the corresponding two ___________ and a side of a second triangle, then the two triangles are _________________.

Abbreviation:

Example #1: Write a two-column proof.

Given: \( AB \) bisects \( \angle CAD \)
\[ \angle 1 \cong \angle 2 \]

Prove: \( \Delta CAB \cong \Delta DAB \)
Example #2: Write a two-column proof.

Given: $\overline{AD} \parallel \overline{CB}$
$\angle A \cong \angle C$

Prove: $\triangle DGB \cong \triangle EGC$

Example #3: Write a two-column proof.

Given: $\angle V \cong \angle S$
$TV \cong QS$

Prove: $\overline{VR} \cong \overline{SR}$